

Inclusive production of charged pions in pC collisions: energy dependence of invariant cross sections

A. Butkevich

Institute for Nuclear Research, Moscow

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- (★) Precise hadron production measurements in p + C collision at energy E=120 GeV are needed for improving calculation of the NuMI neutrino flux.
- (★) Charged pion spectra in p + C interactions were measured in NA61 and NA49 experiments at proton energies 31 and 158 GeV, correspondingly. These data cover kinematic region of interest for charged pion
 $0.02 \leq x_F \leq 0.3$ and $0.1 \leq p_T \leq 0.5(GeV/c)$
whose daughter muon neutrino gives the main contribution to NuMI neutrino flux.
- (★) In this work we study energy dependence of the measured spectra to estimate the pion invariant cross section at proton energy 120 GeV.

NA49 result at the CERN SPS at 158 GeV/c beam momentum

★ The invariant inclusive cross section

$$f(x_F, p_T) = E(x_F, p_T) \frac{d^3\sigma}{dp^3}(x_F, p_T)$$

was measured in p+C collision at 158 GeV/c beam momentum. $x_F = 2p_{||}^* \sqrt{s}$ is defined in nucleon-nucleon cms and p_T is transverse pion momentum.

- ★ The data cover a phase space area $0 \leq p_T \leq 1.8$ GeV/c and $-0.1 \leq x_F \leq 0.5$. At $0 \leq x_F \leq 0.3$ and $0 \leq p_T \leq 0.5$ the statistical error is about $1 \div 3$ %. In the other region it equals of $3 \div 10$ %. The systematic error (rms) of ≈ 3.8 % was obtained. *C. Alt et al. Eur.Phys.J. C49, 897, 2007*
- ★ A two-dimensional interpolation which produces smooth overall x_F and p_T dependencies was developed. The tables of the invariant cross sections $f(x_F, p_T)$ of charged pion production can be used for comparison with other data measured at different x_F and p_T values.

spshadrons.web.cern.ch/spshadrons/pCdata.html

NA61 result at the CERN SPS at 31 GeV/c beam momentum

- ★ Differential charge pions production cross section in the lab system $d\sigma^\pi/dp(\theta, p)$ was measured in p+C interaction, where p and θ are the pion momentum and polar angle.
- ★ The data cover a phase space area $0 \leq \theta \leq 420$ mrad and $0.2 \leq p \leq p_{max}(\theta)$, where $p_{max} = 5 \div 12$ GeV/c is a function of θ . The statistical error of $5 \div 30$ % and the systematic error (rms) of $\approx 6 \div 20$ % depend on the pion angle and momentum. *N. Abgrall et al. PRC 84, 034604, 2011*
- ★ To compare NA49 and NA61 data the NA61 differential cross section $d\sigma/dp$ was transformed into bin-averaged invariant cross section $(d^3\sigma/dp^3)/E$ as a function of (p_{61}, θ) . *M. Kordosky DocDB-7283*

- ★ The bin-averaged invariant cross section

$$E \frac{d^3\sigma}{dp^3} = f(\Delta p_i, \Delta \theta_j) = \Delta p_i F(\Delta p_i, \Delta \theta_j) / J(\Delta p_i, \Delta \theta_j),$$

where

$$F(\Delta p_i, \Delta \theta_j) = \frac{d\sigma}{dp}(\Delta p_i, \Delta \theta_j),$$

is measured cross section and

$$J(\Delta p_i, \Delta \theta_j) = 2\pi \int_{p_i}^{p_{i+1}} \int_{\theta_j}^{\theta_{j+1}} \frac{p^2 dp}{E} d\cos\theta$$

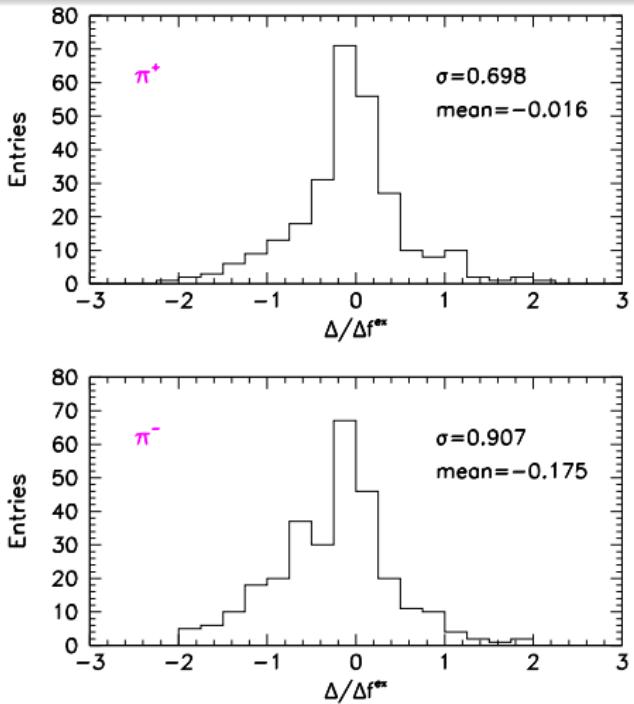
is phase space volume.

- ★ The binning correction was evaluated for $f(p_i, \theta_j)$ data points. The correction is on the 1-4% level.

RESULT

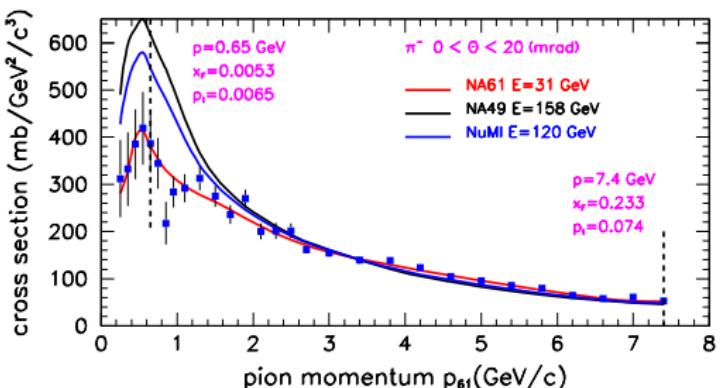
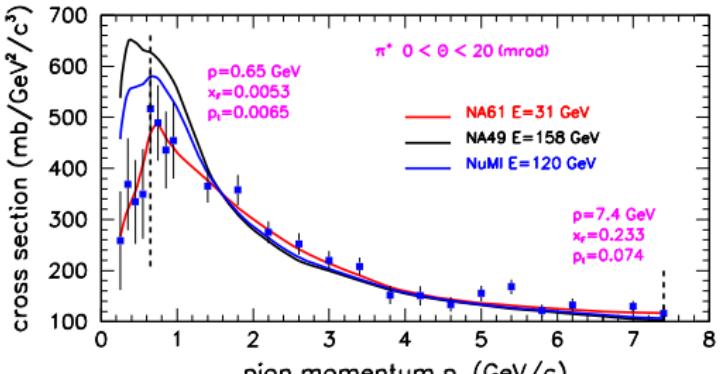
- (*) A two-dimensional interpolation which produces smooth overall p and θ dependencies was applied. It is based on an effective variance recursive method, using manual fit (see BACKUP). The tables of the invariant cross section $f(p_{61}, \theta)$ were obtained.
- (*) The final result can be controlled by evaluating the distribution of the differences between data and interpolated values, divided by the statistical errors of each data point.
- (*) This distribution should be a Gaussian centered at zero and with variance of unity.

Interpolation control



Histograms of the differences Δ -between the measured invariant cross sections and corresponding interpolated values divided by the statistical uncertainty $\sigma(f_{ij})$ of the data points.

- (★) NA61 invariant cross section measured at (p_{61}, θ) corresponds to the cross section measured in NA49 experiment at scaling variables $[x_F(p_{61}, \theta), p_T(p_{61}, \theta)]$.
- (★) We compared NA61 and NA49 interpolated cross sections at the same values (x_F, p_T) .
- (★) The invariant cross sections at proton energy $E_p = 120$ GeV were evaluated using linear fit over energy in the energy range $E_p = 31 \div 158$ GeV for each pair of variables $[x_F(p_{61}, \theta), p_T(p_{61}, \theta)]$.

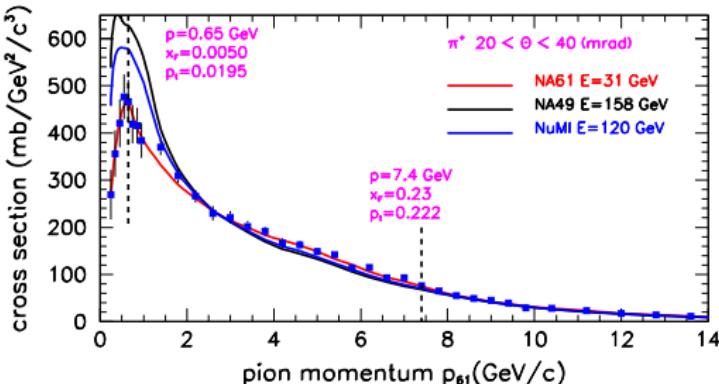


Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 10(\text{mrad})$$

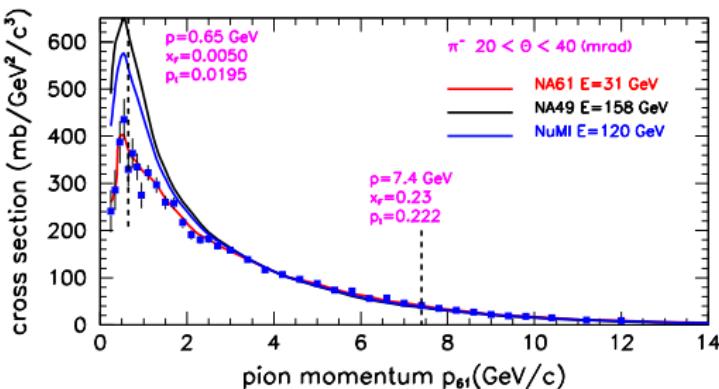
$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$

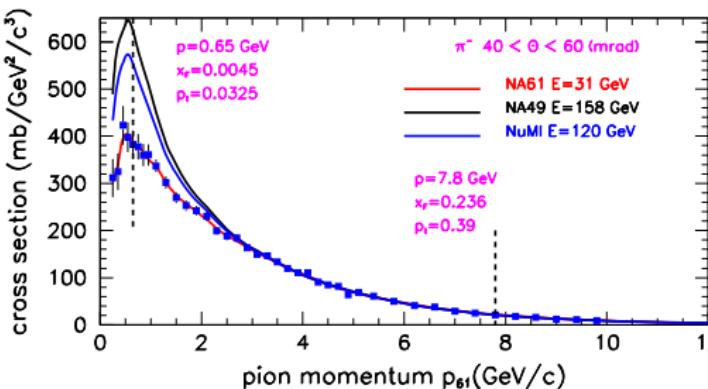
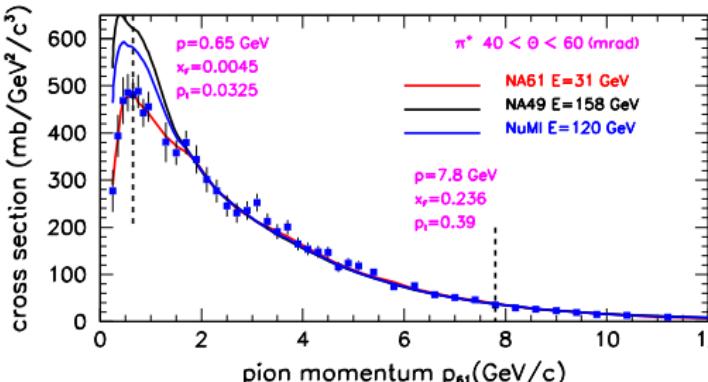
0.25	-2.96^{-2}	2.50^{-3}
0.65	5.31^{-3}	6.50^{-3}
1.40	3.72^{-2}	1.40^{-2}
2.20	6.52^{-2}	2.20^{-2}
3.00	9.18^{-2}	3.00^{-2}
3.80	0.118	4.20^{-2}
4.60	0.144	4.60^{-2}
5.40	0.169	5.40^{-2}
6.20	0.195	6.20^{-2}
7.00	0.221	7.00^{-2}



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 30(mrad)$

$p_{61}(GeV)$	x_F	$p_T(GeV/c)$
0.25	-2.97^{-2}	7.50^{-3}
0.65	5.03^{-3}	1.95^{-2}
1.40	3.67^{-2}	4.20^{-2}
2.20	6.43^{-2}	6.60^{-2}
3.00	9.06^{-2}	9.00^{-2}
3.80	0.116	0.114
4.60	0.142	0.138
5.40	0.167	0.162
6.20	0.192	0.186
7.00	0.218	0.210
8.20	0.255	0.246
9.00	0.281	0.270

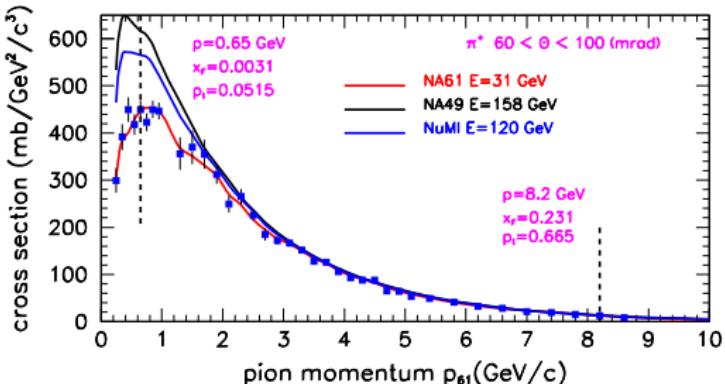




Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 50(\text{mrad})$$

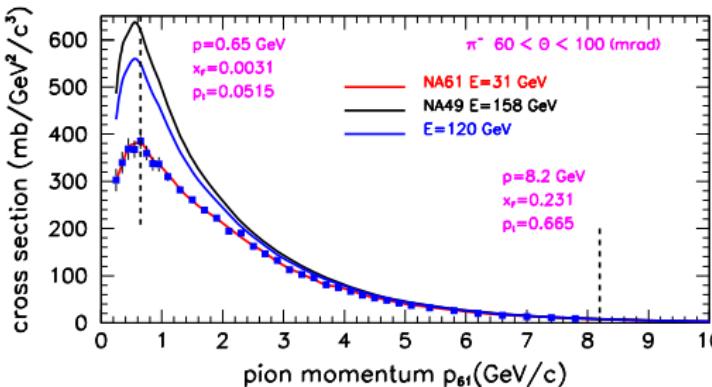
$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-2.99^{-2}	1.25^{-2}
0.65	4.47^{-3}	3.25^{-2}
1.50	3.90^{-2}	7.50^{-2}
2.50	7.21^{-2}	0.125
3.50	0.103	0.174
4.50	0.135	0.225
5.40	0.163	0.270
6.20	0.187	0.310
7.00	0.212	0.350
7.80	0.236	0.389
8.60	0.261	0.430
9.40	0.285	0.470

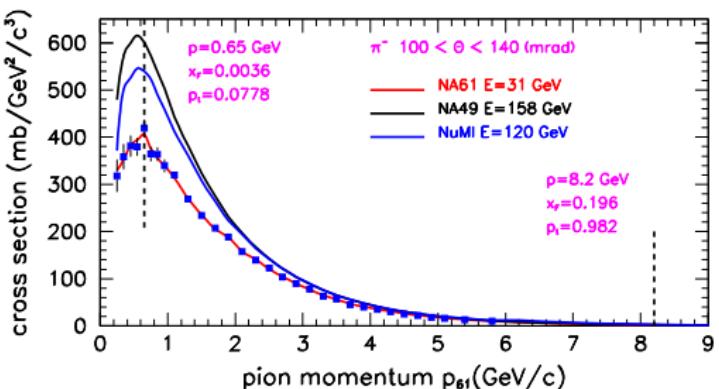
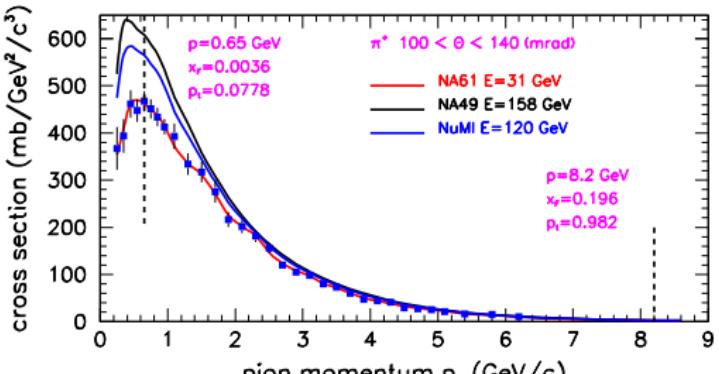


Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 80(\text{mrad})$$

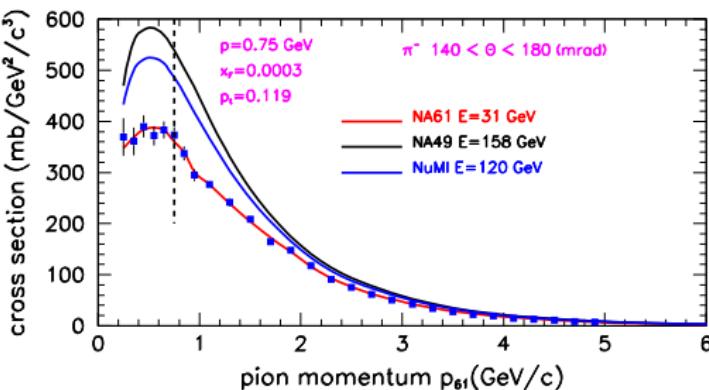
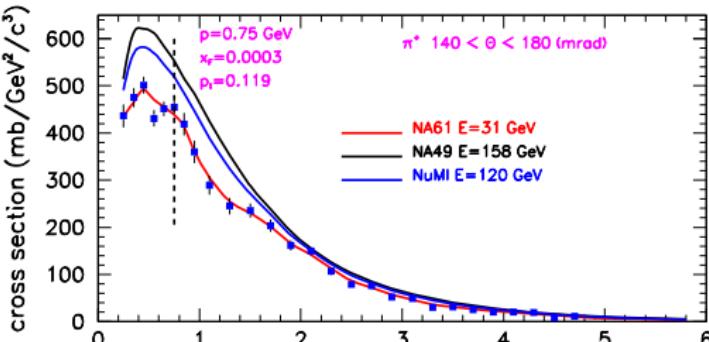
$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-3.05^{-2}	2.00^{-2}
0.65	3.12^{-3}	5.19^{-2}
1.30	2.92^{-2}	0.103
2.30	6.09^{-2}	0.183
3.10	8.47^{-2}	0.248
3.90	0.108	0.312
4.70	0.131	0.376
5.40	0.151	0.432
6.20	0.174	0.495
7.00	0.197	0.559
7.80	0.220	0.623
8.60	0.243	0.687





Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 120(\text{mrad})$

$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-3.15^{-2}	2.99^{-2}
0.65	3.58^{-3}	7.78^{-2}
1.10	1.74^{-2}	0.132
1.50	2.95^{-2}	0.180
2.10	4.50^{-2}	0.251
2.70	6.14^{-2}	0.323
3.50	8.15^{-2}	0.419
4.30	0.101	0.515
5.10	0.121	0.611
5.80	0.138	0.695



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 160 \text{ (mrad)}$$

$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$

0.65 3.51^{-3} 0.104

1.10 1.09^{-2} 0.175

1.50 2.06^{-2} 0.239

2.10 3.34^{-2} 0.335

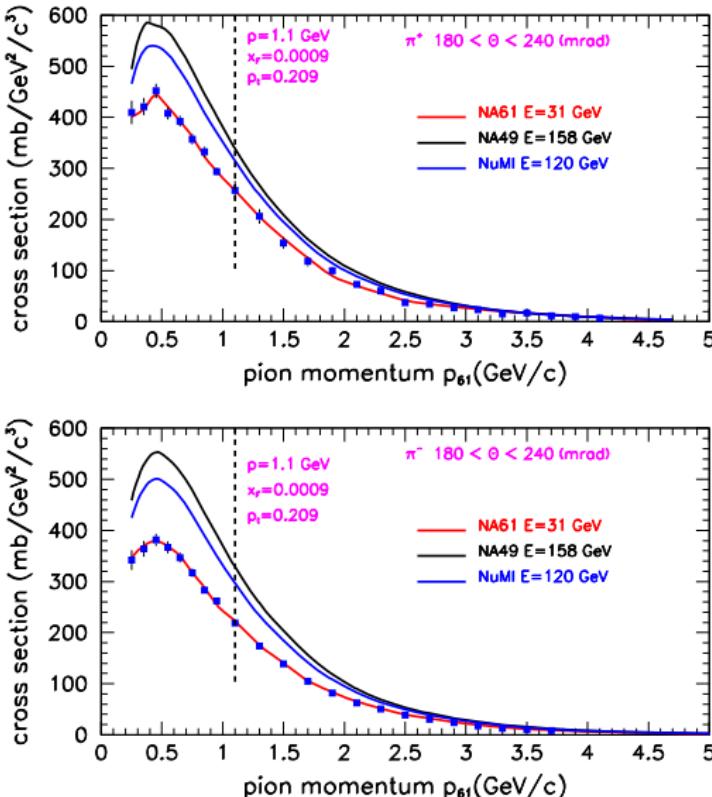
2.70 4.54^{-2} 0.430

3.50 6.07^{-2} 0.559

4.30 7.58^{-2} 0.685

5.10 9.07^{-2} 0.813

5.80 0.104 0.924



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 210(\text{mrad})$$

$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$

0.65 -9.89^{-3} 0.135

1.10 8.79^{-5} 0.229

1.50 5.86^{-3} 0.313

1.90 1.06^{-2} 0.396

2.30 1.49^{-2} 0.479

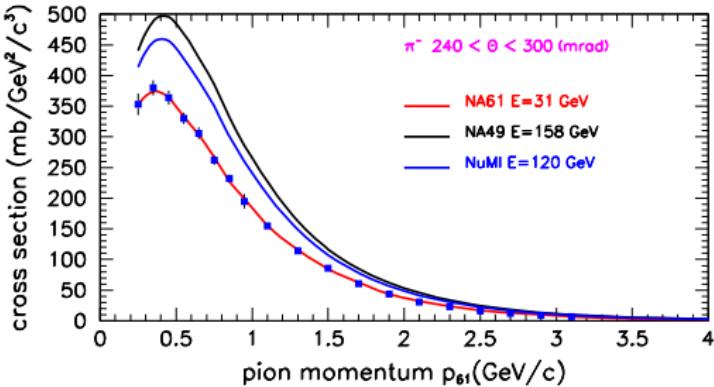
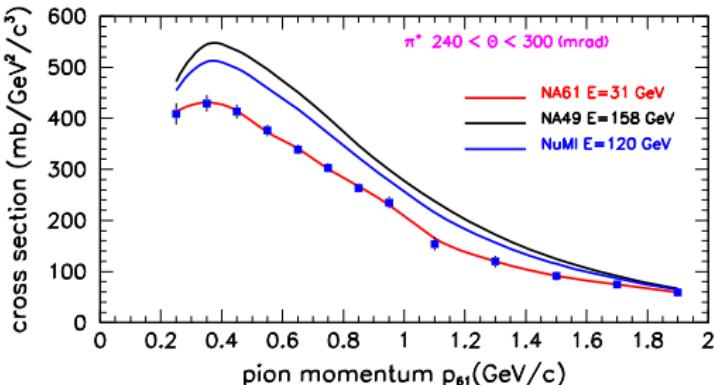
2.70 1.89^{-2} 0.563

3.10 2.27^{-2} 0.646

3.50 2.64^{-2} 0.730

3.90 3.01^{-2} 0.813

4.50 3.54^{-2} 0.938



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 270(\text{mrad})$$

$$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$$

$$0.65 \quad -1.98^{-2} \quad 0.173$$

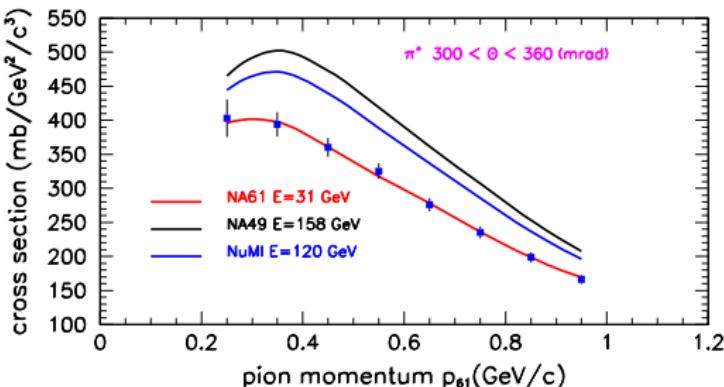
$$0.85 \quad -1.76^{-2} \quad 0.227$$

$$1.10 \quad -1.66^{-2} \quad 0.293$$

$$1.50 \quad -1.69^{-2} \quad 0.400$$

$$1.70 \quad -1.75^{-2} \quad 0.453$$

$$1.80 \quad -1.83^{-2} \quad 0.506$$

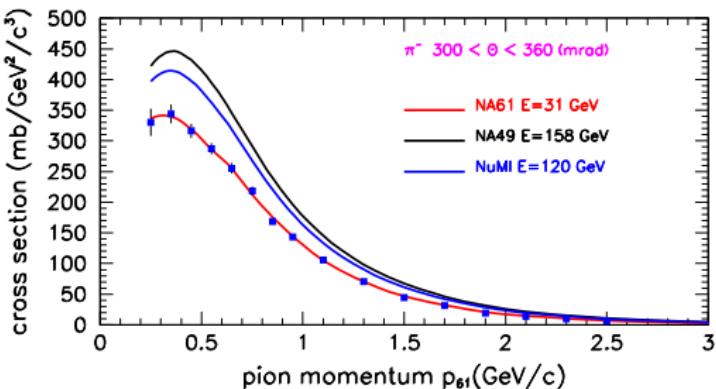


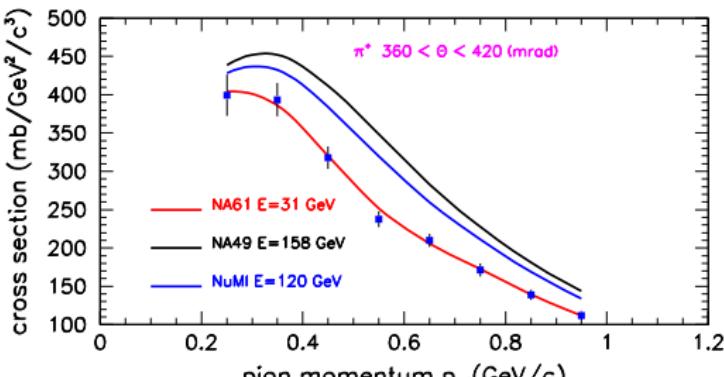
Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 330(mrad)$$

$p_{61}(GeV)$ x_F $p_T(GeV/c)$

0.25	-4.40^{-2}	8.10^{-2}
0.35	-3.67^{-2}	0.113
0.45	-3.34^{-2}	0.146
0.55	-3.22^{-2}	0.178
0.65	-3.20^{-2}	0.216
0.75	-3.26^{-2}	0.243
0.85	-3.36^{-2}	0.275
0.95	-3.50^{-2}	0.308





Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 390(\text{mrad})$$

$p_{61}(GeV) \quad x_F \quad p_T(GeV/c)$

$$0.25 \quad -4.96^{-2} \quad 9.51^{-2}$$

$$0.35 \quad -4.46^{-2} \quad 0.133$$

$$0.45 \quad -4.36^{-2} \quad 0.171$$

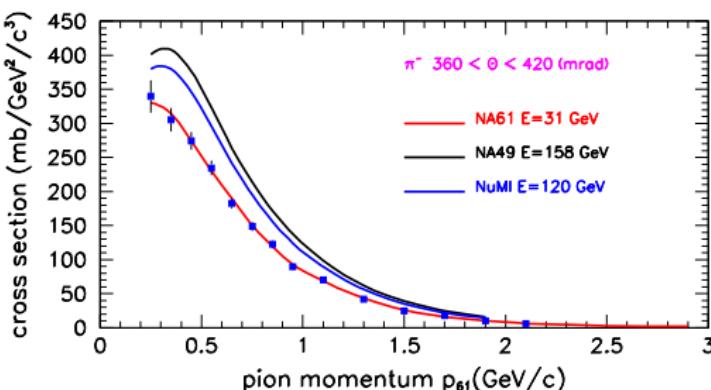
$$0.55 \quad -4.46^{-2} \quad 0.209$$

$$0.65 \quad -4.67^{-2} \quad 0.247$$

$$0.75 \quad -4.95^{-2} \quad 0.285$$

$$0.85 \quad -5.28^{-2} \quad 0.323$$

$$0.95 \quad -5.64^{-2} \quad 0.361$$



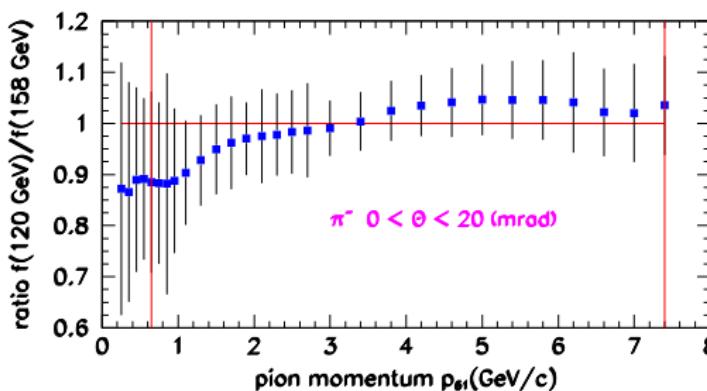
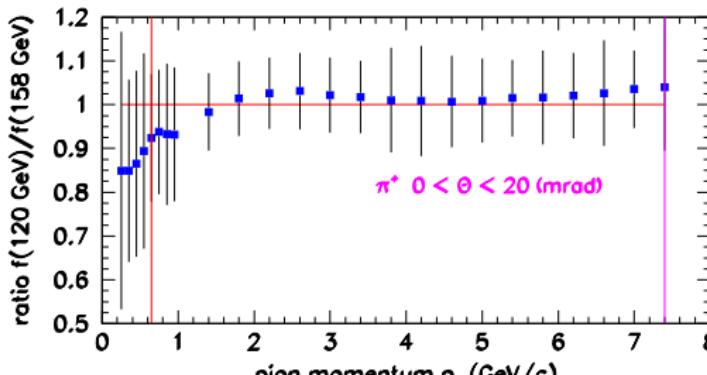
DISCUSSION

We calculated ratio of the invariant cross sections at the same values (x_F, p_T)

$$R = f(p_{61}, \theta, E = 120\text{GeV})/f(p_{61}, \theta, E = 158\text{GeV})$$

as a function of p_{61} , to estimate the “scaling violation” effect in the energy range $E = 120 \div 158$ GeV.

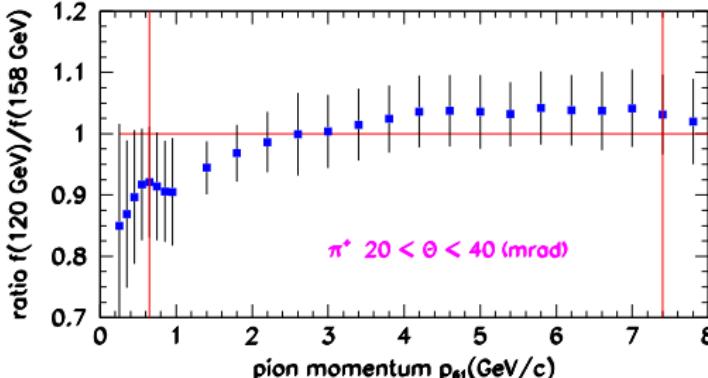
Only the statistical error of NA61 data is shown



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

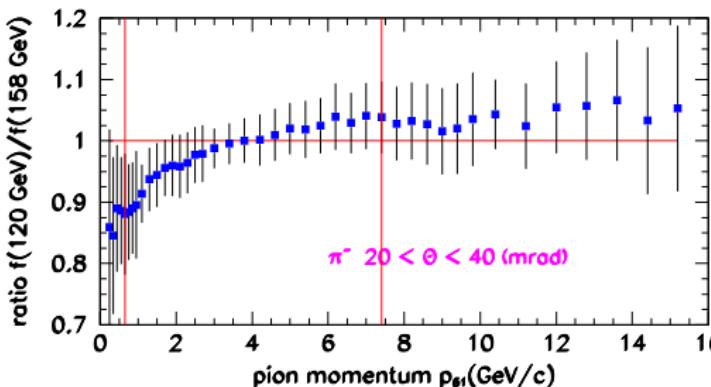
$$\theta = 10 \text{ (mrad)}$$

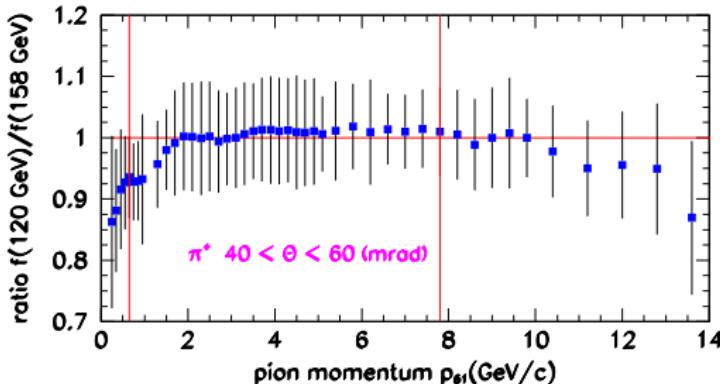
$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-2.96^{-2}	2.50^{-3}
0.65	5.31^{-3}	6.50^{-3}
1.40	3.72^{-2}	1.40^{-2}
2.20	6.52^{-2}	2.20^{-2}
3.00	9.18^{-2}	3.00^{-2}
3.80	0.118	4.20^{-2}
4.60	0.144	4.60^{-2}
5.40	0.169	5.40^{-2}
6.20	0.195	6.20^{-2}
7.00	0.221	7.00^{-2}



Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 30(\text{mrad})$

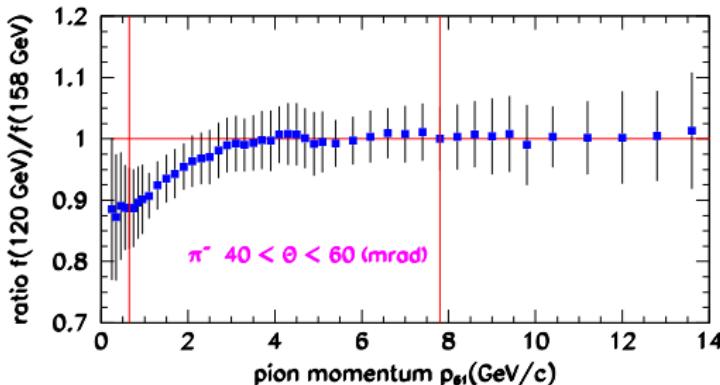
$p_{61} (\text{GeV})$	x_F	$p_T (\text{GeV}/c)$
0.25	-2.97^{-2}	7.50^{-3}
0.65	5.03^{-3}	1.95^{-2}
1.40	3.67^{-2}	4.20^{-2}
2.20	6.43^{-2}	6.60^{-2}
3.00	9.06^{-2}	9.00^{-2}
3.80	0.116	0.114
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5.40	0.167	0.162
6.20	0.192	0.186
7.00	0.218	0.210
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9.00	0.281	0.270

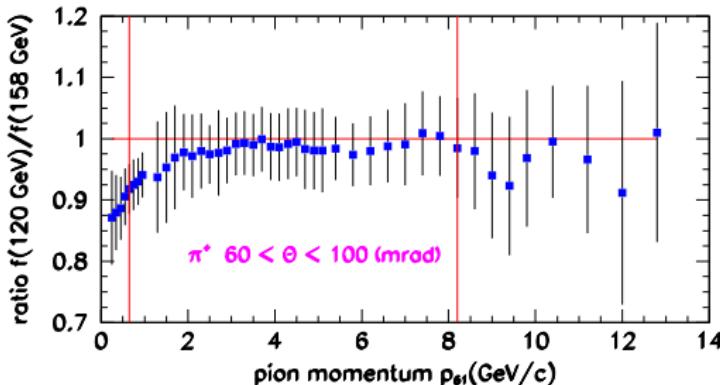




Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 50(\text{mrad})$

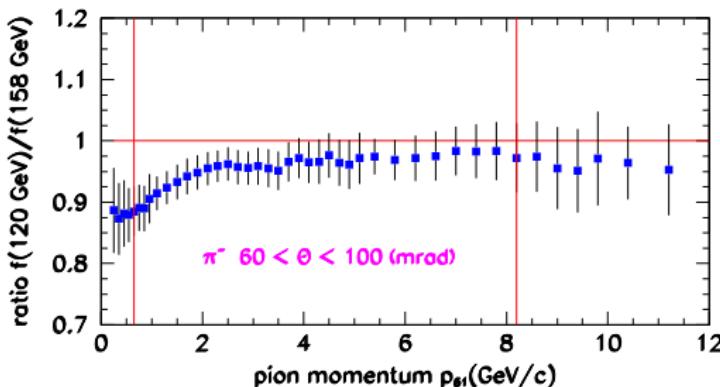
$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-2.99^{-2}	1.25^{-2}
0.65	4.47^{-3}	3.25^{-2}
1.50	3.90^{-2}	7.50^{-2}
2.50	7.21^{-2}	0.125
3.50	0.103	0.174
4.50	0.135	0.225
5.40	0.163	0.270
6.20	0.187	0.310
7.00	0.212	0.350
7.80	0.236	0.389
8.60	0.261	0.430
9.40	0.285	0.470

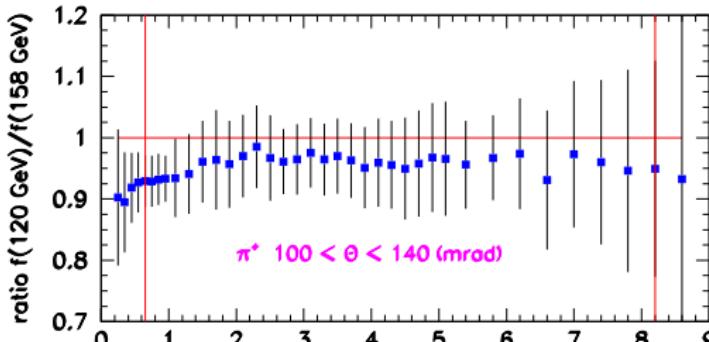




Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 80(\text{mrad})$

$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-3.05^{-2}	2.00^{-2}
0.65	3.12^{-3}	5.19^{-2}
1.30	2.92^{-2}	0.103
2.30	6.09^{-2}	0.183
3.10	8.47^{-2}	0.248
3.90	0.108	0.312
4.70	0.131	0.376
5.40	0.151	0.432
6.20	0.174	0.495
7.00	0.197	0.559
7.80	0.220	0.623
8.60	0.243	0.687





Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 120 \text{ (mrad)}$$

$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$

$$0.25 \quad -3.15^{-2} \quad 2.99^{-2}$$

$$0.65 \quad 3.58^{-3} \quad 7.78^{-2}$$

$$1.10 \quad 1.74^{-2} \quad 0.132$$

$$1.50 \quad 2.95^{-2} \quad 0.180$$

$$2.10 \quad 4.50^{-2} \quad 0.251$$

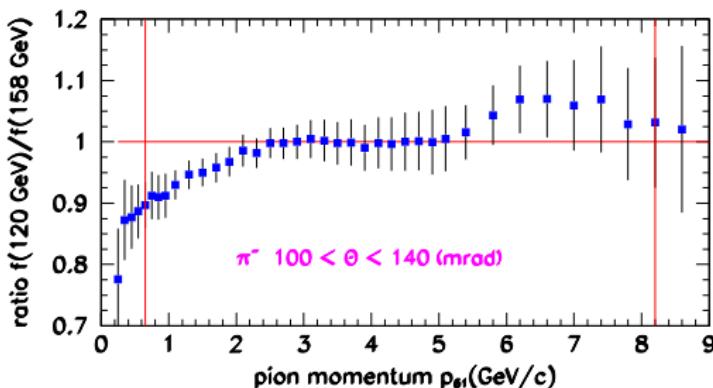
$$2.70 \quad 6.14^{-2} \quad 0.323$$

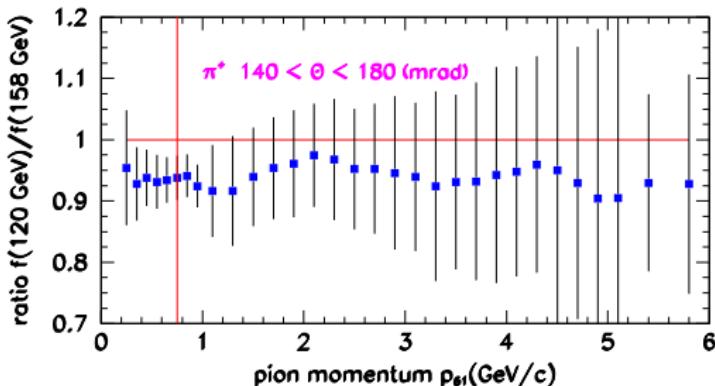
$$3.50 \quad 8.15^{-2} \quad 0.419$$

$$4.30 \quad 0.101 \quad 0.515$$

$$5.10 \quad 0.121 \quad 0.611$$

$$5.80 \quad 0.138 \quad 0.695$$





Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$

$$\theta = 160 \text{ (mrad)}$$

$p_{61}(\text{GeV}) \quad x_F \quad p_T(\text{GeV}/c)$

$0.65 \quad 3.51^{-3} \quad 0.104$

$1.10 \quad 1.09^{-2} \quad 0.175$

$1.50 \quad 2.06^{-2} \quad 0.239$

$2.10 \quad 3.34^{-2} \quad 0.335$

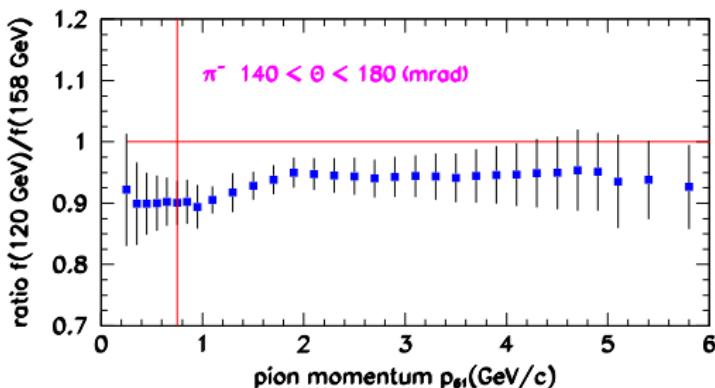
$2.70 \quad 4.54^{-2} \quad 0.430$

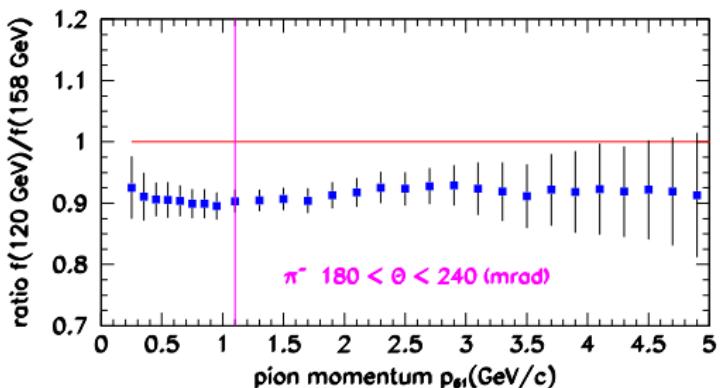
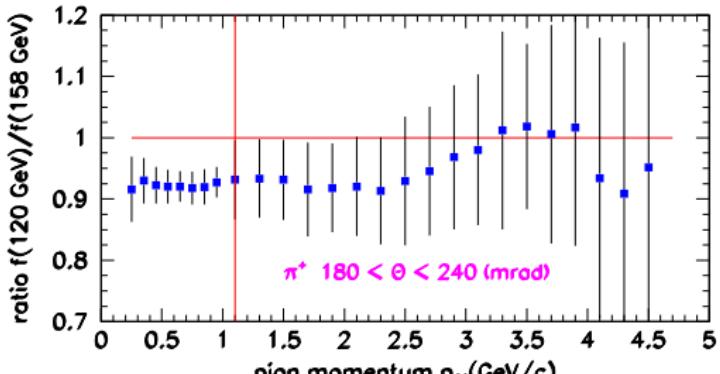
$3.50 \quad 6.07^{-2} \quad 0.559$

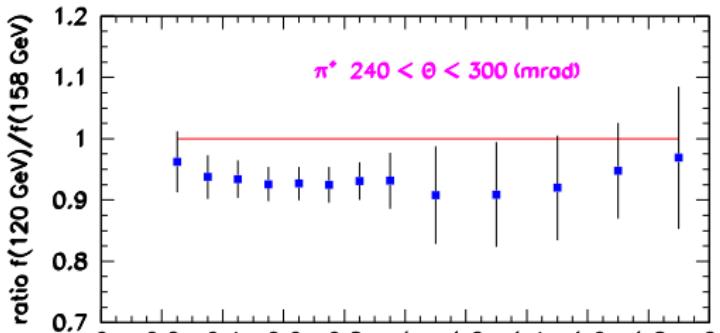
$4.30 \quad 7.58^{-2} \quad 0.685$

$5.10 \quad 9.07^{-2} \quad 0.813$

$5.80 \quad 0.104 \quad 0.924$

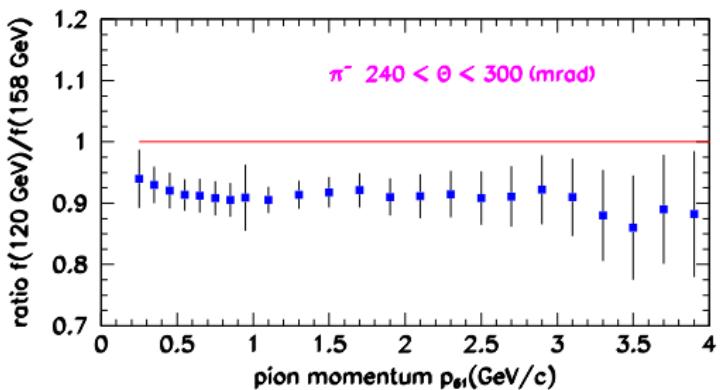


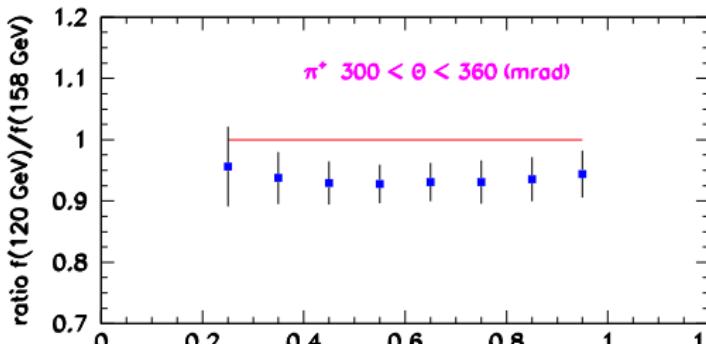




Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 270$ (mrad)

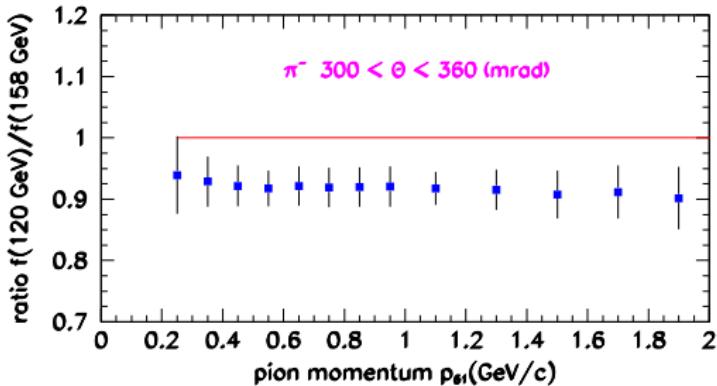
$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.65	-1.98^{-2}	0.173
0.85	-1.76^{-2}	0.227
1.10	-1.66^{-2}	0.293
1.50	-1.69^{-2}	0.400
1.70	-1.75^{-2}	0.453
1.80	-1.83^{-2}	0.506

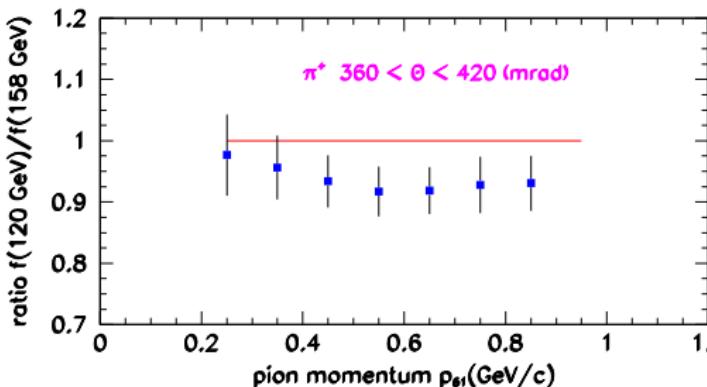




Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 330(\text{mrad})$

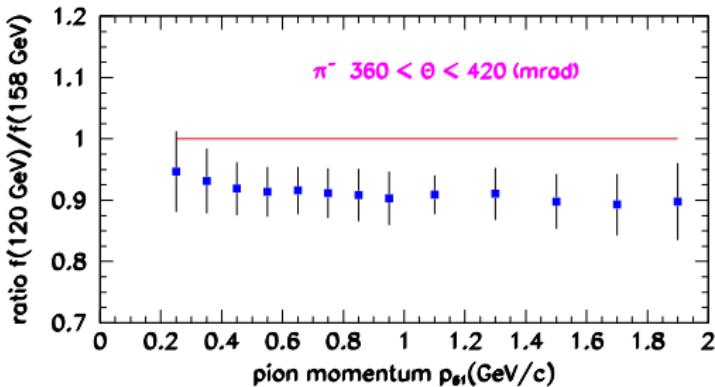
$p_{61}(GeV)$	x_F	$p_T(GeV/c)$
0.25	-4.40^{-2}	8.10^{-2}
0.35	-3.67^{-2}	0.113
0.45	-3.34^{-2}	0.146
0.55	-3.22^{-2}	0.178
0.65	-3.20^{-2}	0.216
0.75	-3.26^{-2}	0.243
0.85	-3.36^{-2}	0.275
0.95	-3.50^{-2}	0.308





Mapping $(\theta, p_{61}) \rightarrow (x_F, p_T)$
 $\theta = 390$ (mrad)

$p_{61}(\text{GeV})$	x_F	$p_T(\text{GeV}/c)$
0.25	-4.96^{-2}	9.51^{-2}
0.35	-4.46^{-2}	0.133
0.45	-4.36^{-2}	0.171
0.55	-4.46^{-2}	0.209
0.65	-4.67^{-2}	0.247
0.75	-4.95^{-2}	0.285
0.85	-5.28^{-2}	0.323
0.95	-5.64^{-2}	0.361



CONCLUSIONS

- (★) The measured NA61 differential cross section $d\sigma/dp$ was transformed into invariant cross section $f(x_F, p_T)$.
- (★) This cross section was interpolated using the effective variance recursive method and compared with interpolated NA49 data at the same values of $[x_F(p_{61}, \theta), p_T(p_{61}, \theta)]$.
- (★) The invariant cross section at proton energy 120 GeV was evaluated (tables) and “scaling violation” effect in the energy range $E = 120 \div 158$ GeV was estimated.
- (★) This effect of $\leq 10\%$ depends on (x_F, p_T) and is of the same order as the statistical errors of NA61 data.
- (★) Note that in this approach the “scaling violation” effect is somewhat overestimated due to liner interpolation over proton energy.

BACKUP

Scaling variables

Scaling variables (x_F, p_T)

$x_F(p, \theta) = p_{||}^* / p_{max}^*$ and $p_T(p, \theta) = p * \sin \theta$, where

$$p_{||}^* = \frac{E_a + m_N}{\sqrt{s}} p_{||} - \frac{p_a}{\sqrt{s}} E$$

$$s = 2m_N(E_a + m_N)$$

$$p_{||} = p \cos \theta$$

$$p_{max}^* = \frac{1}{2s} [(s + m_\pi^2 - s_x^{min})^2 - 4s_x^{min}m_\pi^2]^{1/2},$$

where $s_x^{min} \approx 3.8(4.1)$ GeV² for $\pi^+(\pi^-)$ production, $p_a, E_a(p, E)$ are the proton (pion) momentum and energy, and m_π is mass of pion. At $s \gg s_x^{min} \implies p_{max}^* \simeq \sqrt{s}/2$

Effective variance recursive method

- (★) Invariant cross section $f_{i,j}(p_i, \theta_j)$ with variance $\sigma(f_{ij})$ was measured at N_p points, p_i , and N_θ points θ_j .
- (★) The errors on p_i and θ_j are

$$\sigma_p^2(p_i) = \frac{1}{3}(\Delta p_i/2)^2, \quad \sigma_\theta^2(\theta_j) = \frac{1}{3}(\Delta \theta_j/2)^2,$$

where Δp_i and $\Delta \theta_j$ are momentum and angular bins size.

- (★) The task is to estimate $f(p, \theta)$ for arbitrary (p, θ) , where $p_1 \leq p \leq p_{N_p}$, $\theta_1 \leq \theta \leq \theta_{N_\theta}$, by drawing a smooth curve through the (p_i, θ_j) .
- (★) Two-dimensional second order local polynomial interpolation

$$y(p, \theta) = a_1 + a_2 p + a_3 \theta + a_4 p^2 + a_5 p \theta + a_6 \theta^2$$

that uses a finite number $n_p = 3$ and $n_\theta = 3$ of “nearest neighbor” points is applied.

(*) The best values of a_l are those for which the sum

$$S = \sum_{i,j}^{n_P, n_\theta} [\bar{f}(p_i, \theta_j) - y(p_i, \theta_j)]^2 \omega_{ij}$$

is a minimum. In this approach

$$\bar{f}(p_i, \theta_j) = f_{ij} + \frac{1}{2}y_{pp}(p_i, \theta_j)\sigma^2(p_i) + \frac{1}{2}y_{\theta\theta}(p_i, \theta_j)\sigma^2(\theta_j)$$

and

$$\omega_{ij} = \frac{1}{\bar{\sigma}^2} \sum_{ij} 1/\bar{\sigma}_{ij}^2, \quad \sum_{ij} \omega_{ij} = 1$$

is the weight of the data points.

(*) The effective invariance $\bar{\sigma}_{ij}^2$ can be written as

$$\bar{\sigma}_{ij}^2 = \sigma_{ij}^2 + y_p^2(p_i, \theta_j)\sigma_p^2(p_i) + y_\theta^2(p_i, \theta_j)\sigma_\theta^2(\theta_j),$$

where

(*) derivatives $y_p, y_\theta, y_{pp}, y_{\theta\theta}$ take forms

$$y_p = \partial y / \partial p = a_2 + 2a_4 p_i + a_5 \theta_j$$

$$y_\theta = \partial y / \partial \theta = a_3 + 2a_6 \theta_j + a_5 p_i$$

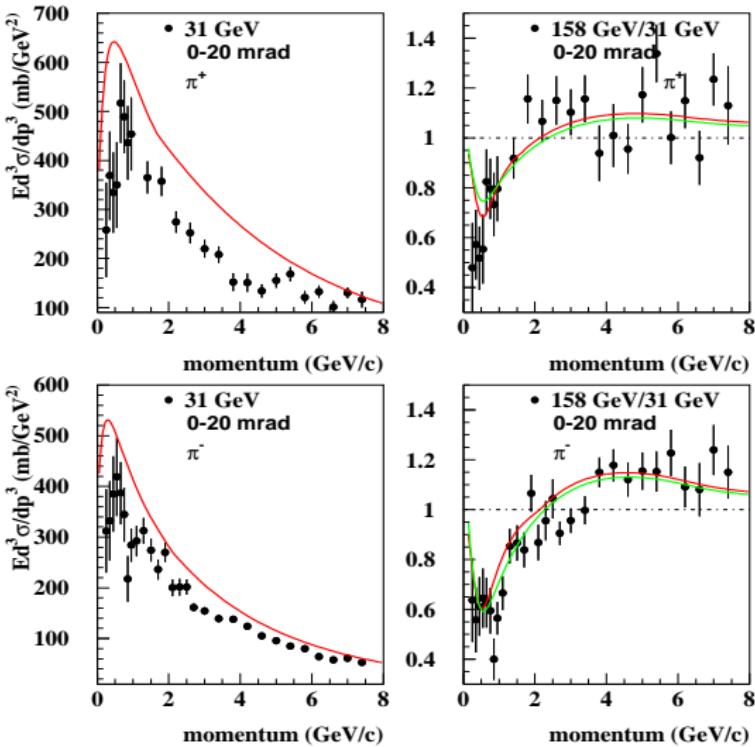
$$y_{pp} = \partial^2 y / \partial p^2 = 2a_4$$

$$y_{\theta\theta} = \partial^2 y / \partial \theta^2 = 2a_6$$

(*) Recursive procedure

at the 1st step $\sigma_P = \sigma_\theta = 0 \implies$ Result $a_l^{(1)}$

at the n-th step the derivatives $f_p, f_\theta, f_{pp}, f_{\theta\theta}$ are evaluated with values of $a_l^{(n-1)}$.



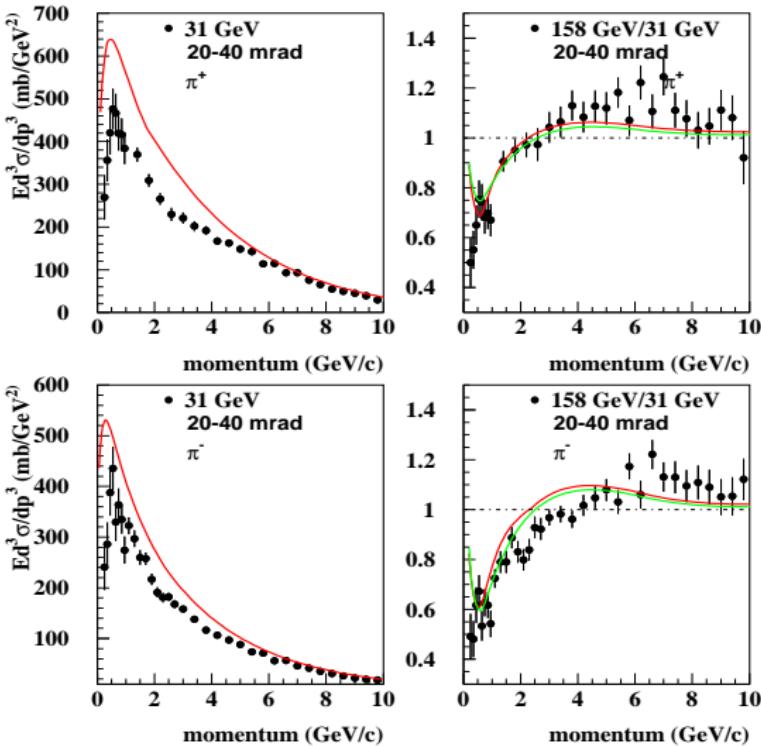
MARS Model (S. Striganov)

N.V. Mokhov et al. (2004)

<http://www-ap.fnal.gov/MARS/>

Green line





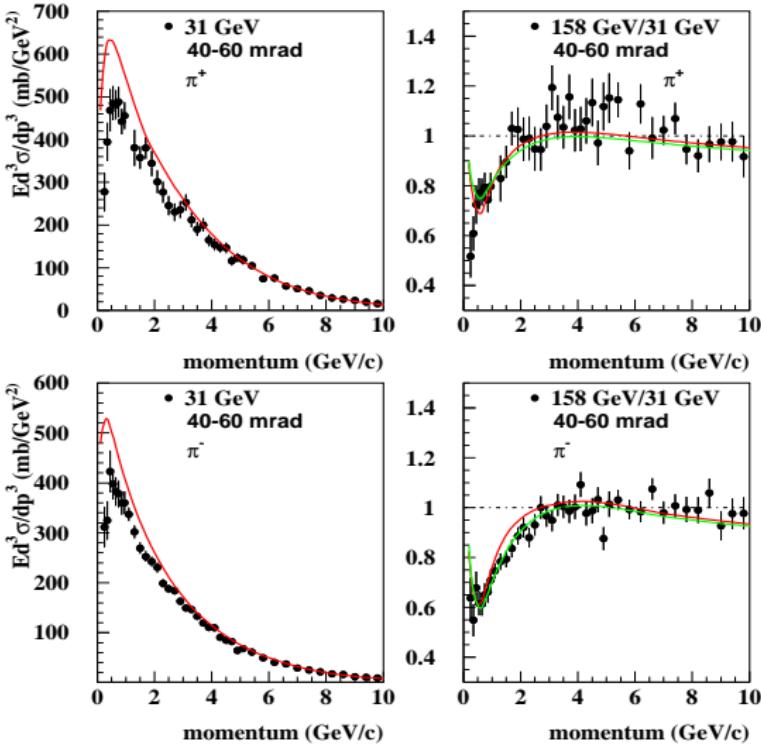
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